Volume Conservation, Dark Matter, Dark Energy, and Gravitation

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Abstract

We propose that through a non-metrizable topology $\mathcal{Q}$, there exists a non-classical inseparability of points on the same spacelike hypersurface. We posit that energy concentrations displace spatial volume to induce a volume-conserving potential across $\mathcal{Q}$, and derive a partial differential equation describing the flow of 3-volume along the potential gradient. From this we make a testable prediction of the cosmological distribution of dark energy and show that the gravitational coupling is proportional to the universe's global capacity for absorbing volumetric flow. We close with a derivation of an upper limit for the universe's total energy, and a conjecture that the volumetric pressure upon 3-space asymptotically approaches the Planck volume as the energy density approaches infinity.

1 Introduction

We begin with the proposition that nature's wave-particle duality[1–4] reflects an intrinsic property of its topological structure, wherein the points of space exist on two topologies simultaneously. This bitopology consists of the familiar metrizable 3-space manifold of our daily experience conjoined with an indiscrete topology on which all points are inseparable.

2 Topological Framework

We take inspiration from the BSSN formalism[5, 6] and foliate the 4-dimensional manifold of spacetime $\mathcal{M}$ into a family of spacelike hypersurfaces $\{\Sigma_t\}$ (the right side of fig. 1),

$$\mathcal{M} = \{\Sigma_t\}. \tag{1}$$

Let Σ_t be one such hypersurface and γ_{ij} the spatial metric projected on Σ_t from $\mathcal{M}$'s metric tensor, $g_{\mu\nu}$. In order to decouple our measure for integration from the metric geometry, support non-orientable topologies like those in [7, 8], and keep the units of volume on the metric-free side of the framework (where we intend to build our volume conservation mechanics), we diverge from the BSSN formalism in the metric decomposition. We decompose γ_{ij} into a smooth strictly positive scalar 1-density Ω_t with units of volume, and a unimodular conformal tensor density $\bar{\gamma}_{ij}$ which lacks units and encapsulates only geometric structure.

Ω_t scales with γ_{ij}'s determinant such that

$$\Omega_t^2 = |\gamma|, \tag{2}$$

while the tensor density is made invariant with respect to volume transformations through the inverse cube root of the determinant

$$\bar{\gamma}_{ij} = \frac{1}{\sqrt[3]{|\gamma|}}\,\gamma_{ij}, \tag{3}$$

which gives the direct relation

$$\gamma_{ij} = \Omega_t^{2/3}\,\bar{\gamma}_{ij}. \tag{4}$$

Since volume density is the more foundational topological structure in this framework, we propose that volume is nature's fundamental unit of geometric scale, and that units of the tensor density's spatial axes derive scale from Ω_t according to $\sqrt[3]{\Omega_t}$ or, if considering arbitrary spatial dimensions, $\sqrt[n]{\Omega_t}$.

2.1 Non-Classical Pregeometry

From the components of this decomposition we now rebuild the geometric structure of spacetime from a pregeometric topological foundation akin to Wheeler's propositional pregeometry[9, 10], though in our case more set theoretic than propositional.

We define two metric-free structures conjoined in bitopology over the points of the spacelike hypersurface, Σ_t. The first is the 3-dimensional manifold $\mathcal{V}$, induced from Σ_t through

$$\Sigma_t = \left(\mathcal{V}\left[\Omega_t\right], \bar{\gamma}_{ij}\right), \tag{5}$$

and thus equipped with the volume measure Ω_t from eq. (2).

As defined, $\mathcal{V}$ is a bare manifold that contains Σ_t's smooth structure and underlying topology τ_Σ of open sets, but lacks $\bar{\gamma}_{ij}$'s conformal geometric structure. Specifically, for any region $\mathcal{R} \subseteq \mathcal{V}$, $\mathcal{R}$'s volume is given by $\int_{\mathcal{R}} \Omega_t$, but, lacking

$\bar{\gamma}_{ij}$, there exists no measure for orientation, distance, angles or area on $\mathcal{V}$. When operating strictly within the structure of $\mathcal{V}$'s bare manifold, we will use the language of differential forms and refer to Ω_t as a twisted 3-form[11, 12].

For the conjoined structure of our bitopology, we use the unary union $\bigcup$ to take the points from $\mathcal{V}$'s topology of open sets,

$$\mathbb{S} = \bigcup \tau_\Sigma = \{x \mid x \in U \text{ for some open set } U \in \tau_\Sigma\}, \tag{6}$$

and construct an indiscrete topology $\mathcal{Q}$ in which the only non-empty open set is the set of all points of $\mathbb{S}$. Where there is no risk of ambiguity, we will hereafter abuse notation and use $\mathcal{V}$ as shorthand for the induced topology τ_Σ.

To aid intuition for this conjoined structural pair of $\mathcal{V}$ and $\mathcal{Q}$, encode a relationship between these structures by defining the degenerate product on $\mathcal{V}$'s tangent bundle by

$$\langle u, v \rangle_{\mathcal{Q}} = 0, \; \forall \; u, v \in T_p\mathcal{V} \;\text{ and }\; p \in \mathcal{V}\,. \tag{7}$$

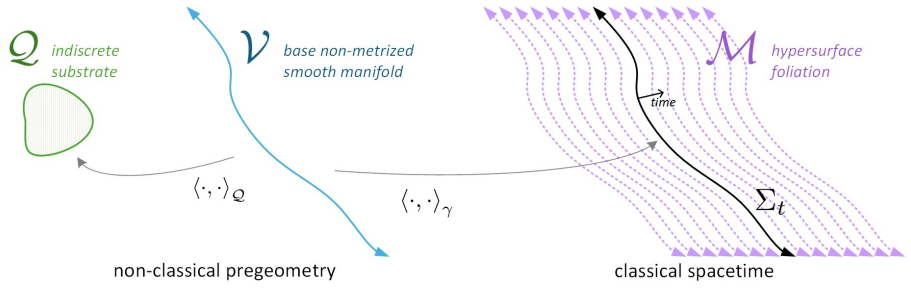


Figure 1: Bitopological Framework

Let the full topological structure on the points of space be the bitopology $\mathcal{U} \equiv (\mathbb{S}, \tau_\Sigma, \mathcal{Q})$, or, leaving $\mathbb{S}$ implied and using the aforementioned shorthand,

$$\mathcal{U} = (\mathcal{V}, \mathcal{Q})\,. \tag{8}$$

When the tensor density's geometric structure is required, we replace $\mathcal{V}$ with eq. (5)'s Σ and define $\Upsilon \equiv (\Sigma, \mathcal{Q})$. We let $\mathcal{M}$ be a family of $\{\Upsilon_t\}$, then stitch the spacelike foliation back together with

$$\mathcal{M} = \left\{ \left(\left(\mathcal{V}\left[\Omega_t\right], \bar{\gamma}_{ij} \right), \mathcal{Q}_t \right) \right\}, \tag{9}$$

visualizing $\mathcal{M}$ as a sequence of $(\Sigma_t, \mathcal{Q}_t)$ progressing in successive epochs of cosmic time.

2.2 Classical Law

With this adjustment to $\mathcal{M}$'s definition, we posit that non-local quantum entanglement[13–17], tunnelling[18–20], and wave function measurement effects—first framed as a paradox by Einstein, Podolsky, and Rosen[21, 22] yet shown to be inevitable by Bell[23]—avoid violation of relativistic locality by allowing non-classical mechanics through $\mathcal{Q}$'s inseparability.

We define non-classical mechanics as those invariant with respect to $\bar{\gamma}_{ij}$. The bitopology $(\mathcal{V}, \mathcal{Q})$ from eq. (8) is the natural space for analyzing such mechanics, where freedom from the constraints of the metric enables effects to operate through $\mathcal{Q}$'s indiscrete connectivity to points anywhere on the same spacelike hypersurface.

Conversely, classical mechanics are those dependent on a geometric measure. Classical effects are subject to the constraints of Σ's $\bar{\gamma}_{ij}$ in eq. (5). For these effects, connectivity through $\mathcal{Q}$ is impassable, as a non-metrizable topology is fundamentally incapable of providing the necessary metric.

We use this definition of a non-classical mechanic to build our volume conservation proposal into an experimentally testable hypothesis. A deeper exploration of the implications to quantum and classical theory is a topic for future work.

2.3 Volume Conservation

We propose that $\mathcal{V}$ has constant volume

$$\mathbb{V} = \int_{\mathcal{V}} \Omega_t \, , \forall \, t. \tag{10}$$

Propose $\mathcal{V}$ to have a non-classical volume conservation mechanism, which therefore has no dependence on $\bar{\gamma}_{ij}$. When volumetric pressure requires local deformations in Ω_t to reassign elsewhere on $\mathcal{V}$, the effect operates through the same topology $\mathcal{Q}$ that governs quantum measurement, tunneling, and entanglement.

As unintuitive as a volume conservation mechanism that reassigns units of volume across spacelike intervals may seem, there is related theoretical precedent. A non-metrizable $\mathcal{Q}$ as the enabler of such behavior has explanatory power comparable to that of a wormhole[24] but through proposing an exotic topology rather than exotic spacetime curvature. Though before developing our volume conservation mechanic's formal structure, we present a motivational thought experiment.

3 Thought Experiment

Start with a vacuum spacetime into which we insert a sphere of energy E with density ρ and radius r at point x^i (fig. 2). Following the series expansions in [25],

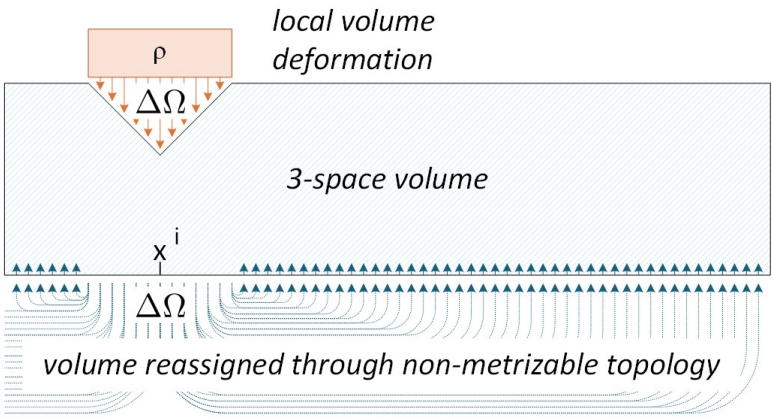


Figure 2: Volume Displacement

our existing relativity theory expects the energy density around x^i to compress the local volume according to

$$\Delta\Omega \propto -\frac{8\pi r^2 G}{15c^2}\,\rho + \mathcal{O}(r^4)\,. \tag{11}$$

For our thought experiment, with volume conserved, this displaced volume must be offset with a corresponding increase elsewhere in x^i's spatial slice, as shown in (fig. 2).

3.1 Residual Stress

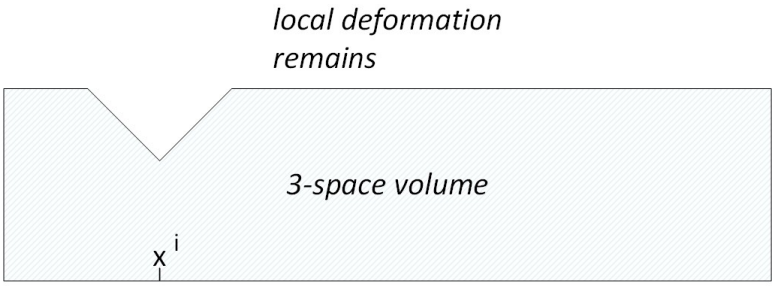


Figure 3: Residual Stress

Imagine some time to pass, remove E, and consider what happens to the stress energy tensor. We would normally observe that the contracted Bianchi identities, $\nabla_\mu G^{\mu\nu} = 0$, require $\nabla_\mu T^{\mu\nu} \to 0$ to conserve energy locally. But in our thought experiment, since this requires that volume around x^i increase to its previous

state, we have a contradiction. The volume has already been reassigned globally to preserve $\mathbb{V}$ and thus, the previous volume cannot be so easily restored (fig. 3). What occurs instead is a form of spacetime plasticity reminiscent of that suggested by Feoli[26], where the Bianchi identities require that residual non-zero stress remain amongst the stress-energy terms to offset the unrestored volume.

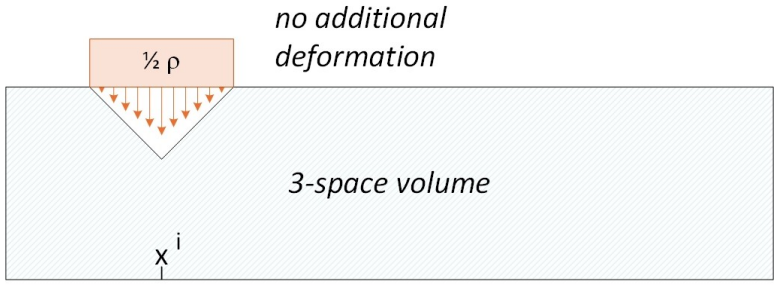


Figure 4: Dark Matter

Now imagine inserting a unit of energy $E_{1/2}$ at x^i that is equivalent to E except that its energy density is $\frac{\rho}{2}$. Inserting $\rho/2$ into eq. (11) gives the volume pressure from $E_{1/2}$ at x^i,

$$\Delta\Omega \propto -\frac{1}{2}\cdot\frac{8\pi r^2 G}{15c^2}\,\rho + \mathcal{O}(r^4)\,. \tag{12}$$

Since this is strictly less than the volume deformation left behind by E from eq. (11), $E_{1/2}$ will settle into E's residual gravity well with no additional deformation (fig. 4).

3.2 Global Smoothing

Finally, imagine reinserting E at some y^i such that the separation between x^i and y^i on γ_{ij} is greater than $2r$. A local deformation of volume will form at y^i, and the displaced volume will reassign across y^i's spatial slice to preserve $\mathbb{V}$.

Most importantly, residual stress around x^i will reduce (fig. 5).

4 Volumetric Potential

Let ζ be a strictly positive and strictly monotonically decreasing smooth function on $\mathbb{R}_{\geq 0}$ (fig. 6) which represents the universe's relationship between energy density and volume. By definition, ζ satisfies $\text{Image}(\zeta) \subset (0, \zeta(0)]$. Let $\rho \in C^\infty(\mathcal{V})$ be the 0-form defining the energy density on $\mathcal{V}$, and interpret $(\zeta \circ \rho)\,(q)$ to be the maximum volume possible at each point $q \in \mathcal{V}$.

We call ζ the **energy-volume relation**, and $\zeta(0)$ the **vacuum volume limit**.

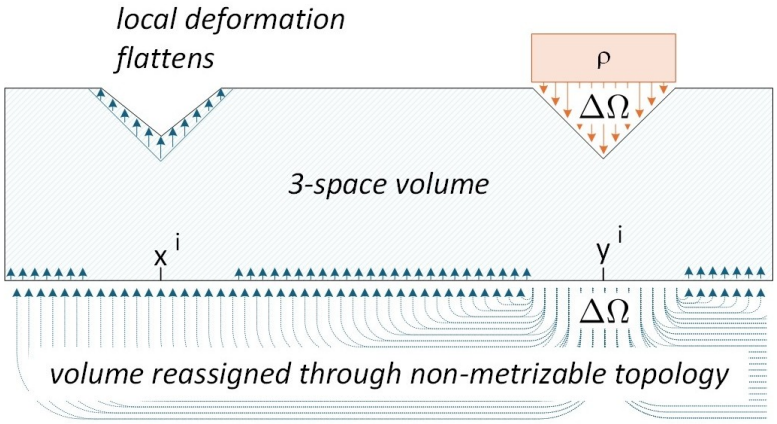


Figure 5: Dark Energy

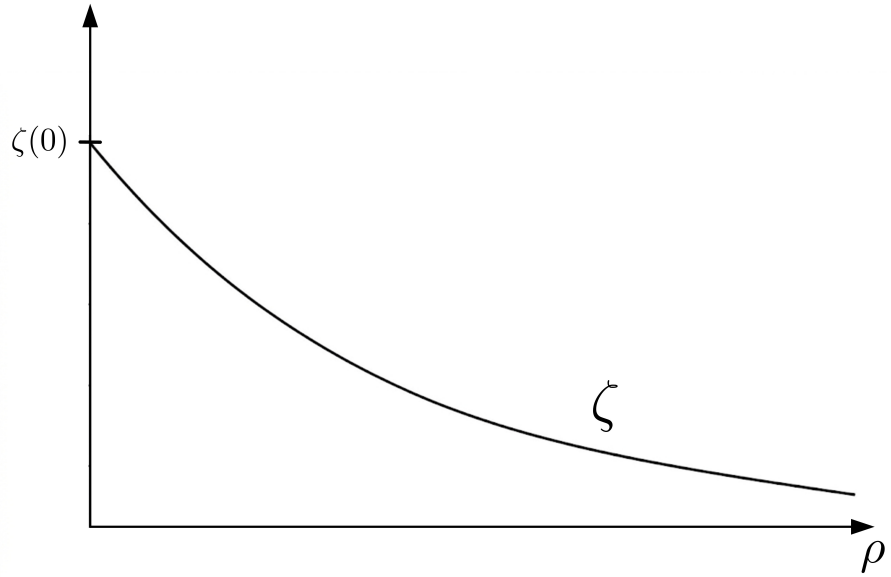


Figure 6: Energy-volume Relation

ζ's smoothness means that $\zeta \circ \rho$ is also a 0-form (i.e., $\zeta \circ \rho \in C^\infty(\mathcal{V})$). With our 3-dimensional $\mathcal{V}$, we know that the space of 0-forms on $\mathcal{V}$ and the space of twisted 3-forms on $\mathcal{V}$ (which we write as $\Gamma(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V})$) are isomorphic line bundles over $\mathcal{V}$, and that with any choice of nowhere-vanishing twisted 3-form, we can generate a one-to-one correspondence.

We pick an arbitrary dimensionless non-vanishing $\omega_0 \in \Gamma(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V})$ to be the generator for our isomorphism, and define

$$\imath : C^\infty(\mathcal{V}) \xrightarrow{\sim} \Gamma\big(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V}\big), \tag{13}$$

by $\imath(f) = f\omega_0$ for each $f \in C^\infty(\mathcal{V})$.

4.1 Top-form Operator Notation

We now define the operators $\sharp$ and $\flat$ to simplify our notation when transitioning physical quantities between their 0-form and 3-form representations.

For a smooth scalar function $f \in C^\infty(\mathcal{V})$, we define the **sharp operator**, $\sharp : C^\infty(\mathcal{V}) \longrightarrow \Gamma(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V})$, by

$$^\sharp f \equiv \imath(f) = f\omega_0\,, \tag{14}$$

and refer to $^\sharp f$ as f **sharp**. For a twisted 3-form $F \in \Gamma(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V})$, we define the **flat operator**, $\flat : \Gamma(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V}) \longrightarrow C^\infty(\mathcal{V})$, by

$$_\flat F \equiv \imath^{-1}(F) = \frac{F}{\omega_0}\,, \tag{15}$$

and refer to $_\flat F$ as F **flat**. Note that by definition, $_\flat(^\sharp f) = f$ and $^\sharp(_\flat F) = F$.

We will regularly make use of the fact that division of a top-form by a non-vanishing top-form is defined and produces a 0-form (as we did in eq. (15)). Specifically, since $\mathcal{V}$'s 3-forms are top-forms, it follows that if A and$B$ are 3-forms from $\Gamma(\Lambda^3 T^*\mathcal{V} \otimes \mathcal{O}_\mathcal{V})$, and B is non-vanishing, then

$$\frac{A}{B} = \frac{_\flat A}{_\flat B} \in C^\infty(\mathcal{V})\,. \tag{16}$$

4.2 Volumetric Limit

We define the twisted 3-form

$$\Omega_\rho \equiv {}^\sharp\big(\zeta \circ \rho\big) \tag{17}$$

to be the **volumetric limit**, and let its upper limit,

$$\Omega_{\max} \equiv {}^{\sharp}\zeta(0)\,, \tag{18}$$

be the **vacuum volume limit**'s twisted 3-form expression.

4.3 Volumetric Gradient

In a region where Ω_ρ drops below the local volume Ω_t, the volumetric pressure within the region will reduce Ω_t to Ω_ρ by causing $\Omega_t - \Omega_\rho$ units of volume to flow out.

An integral across all such regions yields the **volumetric gradient** for a spacelike foliation Σ_t,

$$\Delta\Omega = -\int_{\mathcal{V}_-} (\Omega_\rho - \Omega_t)\,, \tag{19}$$

where $\mathcal{V}_- \equiv \{\, q \in \mathcal{V} \,|\, \Omega_t(q) > \Omega_\rho(q) \,\}$ (fig. 7).

Define the **volumetric potential** to be the twisted 3-form

$$\Phi \equiv \Omega_\rho - \Omega_t\,, \tag{20}$$

which has the scalar expression ${}_\flat\Phi$. Then complete the partition of $\mathcal{V}$ with $\mathcal{V}_+ \equiv \{\, q \in \mathcal{V} \,|\, \Omega_t(q) < \Omega_\rho(q) \,\}$ and $\mathcal{V}_0 \equiv \{\, q \in \mathcal{V} \,|\, \Omega_t(q) = \Omega_\rho(q) \,\}$, as shown in fig. 7.

Then we can write

$$\mathcal{V} = \mathcal{V}_- \cup \mathcal{V}_+ \cup \mathcal{V}_0, \tag{21}$$

and summarize our partition of $\mathcal{V}$ succinctly:

- $\mathcal{V}_-$ is the set of all points where $\Phi < 0$. Volume here exceeds the volumetric limit from eq. (17), inducing a volumetric gradient that directs flow away according to eq. (19).
- $\mathcal{V}_+$ is the set of all points where $\Phi > 0$. Here, with volume below the volumetric limit, there is excess volumetric potential. The volumetric gradient flows toward these regions with a magnitude proportional to the product of eq. (19) with a probability distribution W that varies proportional to ${}_\flat\Phi$. We will construct W in section 4.4.
- $\mathcal{V}_0$ is the set of all points where $\Phi = 0$. Volume here is at its limit. No volume is flowing away and no volume is flowing in.

We propose that the volumetric flow follows eq. (19)'s gradient from the universe's $\mathcal{V}_-$ partition to its $\mathcal{V}_+$ partition (fig. 7) as instantaneously as the Planck time

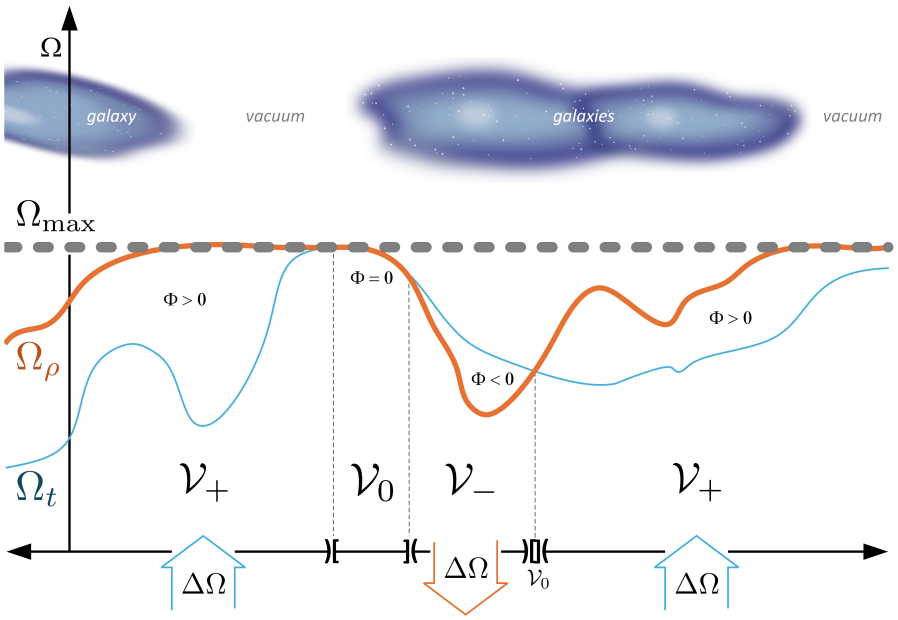


Figure 7: Volumetric Potential

allows, with volume continuously tunneling across spacelike separations through $\mathcal{Q}$. This yields

$$\frac{\Delta\Omega}{\Delta t} = -\frac{1}{t_{\mathrm{P}}}\int_{\mathcal{V}_-}(\Omega_\rho - \Omega_t)\,. \tag{22}$$

Note that this flow does not require anything to be entangled. We only hypothesize that the volumetric potential's gradient is applied across the same non-metrizable topology through which entanglement is maintained, and that this alone is sufficient to enable the tunneling of displaced volume.

For each $q \in \mathcal{V}_-$, the negative outflow is thus $\Phi \,/\, t_{\mathrm{P}}$, and for each $q \in \mathcal{V}_+$, the positive inflow is

$$\frac{(W \circ \flat\Phi)}{\int_{\mathcal{V}_+} \sharp(W \circ \flat\Phi)}\frac{1}{t_{\mathrm{P}}}\int_{\mathcal{V}_-}(-\Phi)\,, \tag{23}$$

where W is a strictly positive smooth function on $\mathbb{R}_{>0}$ that weights the distribution of the flow. For shorthand we will use Φ_W to represent the scalar weight distribution of the twisted 3-form Φ,

$$\Phi_W \equiv W \circ \flat\Phi. \tag{24}$$

4.3.1 Convergence

We will cover in section 5.5.1 that the ultimate verification of convergence for both integrals in eq. (23) is the experimental observation of volumetric flow. Here, before deriving W, we consider the properties $\mathcal{V}_-$ and $\mathcal{V}_+$ must possess for these integrals to converge.

The first point of potential divergence is the limit where the volumetric gradient from eq. (19) is unbounded. Since this is an integral of $(-\Phi)$ across all points where $\Phi < 0$, and since $|\Phi| < {}^\sharp(\ell_{\mathrm{P}}^3) \implies \Phi = 0$, we can assume that across $\mathcal{V}_-$, $|\Phi| \geq {}^\sharp(\ell_{\mathrm{P}}^3)$. This strict non-zero lower bound for $(-\Phi)$ across the volume demonstrates that the integral of $(-\Phi)$ over this volume cannot approach a limit of zero if the volume has infinite extent and therefore cannot converge unless $\mathcal{V}_-$ is compact. Furthermore, we can show that every point of $\mathcal{V}_-$ must have non-zero energy density if we borrow an equation from section 5.

Proof: In eq. (38) we see that if Ω_t ever goes above $\Omega_{\max}$, it will drop back down to $\Omega_{\max}$ within a $\Delta t = 5.39 \times 10^{-44}$ seconds (the Planck time, t_{P}). The same is true if Ω_t ever goes above Ω_ρ. As a result, since all points in $\mathcal{V}_-$ have the property that $\Omega_t > \Omega_\rho$, we can conclude that Ω_ρ must be strictly less than $\Omega_{\max}$ across $\mathcal{V}_-$. Thus, since ζ's strict monotonicity ensures $\Omega_\rho = \Omega_{\max}$ only when $\rho = 0$, we conclude that all points in $\mathcal{V}_-$ must have $\rho > 0$. □

From this it follows that $\mathcal{V}_-$ is non-compact if and only if there is enough energy in the universe to cover a non-compact volume, and hence that eq. (19) can diverge only if both sides of the relation $\mathcal{V}_- \subseteq \mathcal{V}$ are non-compact, and the universe's total energy $\mathbb{E}$ is infinite.

The second point of potential divergence is the limit where the integral in the denominator goes to zero. Since Φ_W from eq. (24) is strictly positive across $\mathcal{V}_+$, we know that $\int_{\mathcal{V}_+} {}^\sharp\Phi_W = 0$ only if $\mathcal{V}_+ = \emptyset$. This represents a universe with no capacity to receive volumetric flow. If this were possible, the volume of 3-space would be infinitely rigid and unable to deform in response to energy. We thus know this integral to be non-zero since it is empirically well established that space does bend in the presence of energy[27, 28].

4.4 Volumetric Flow Distribution

The volumetric potential Φ is a twisted 3-form with $\Phi(\mathcal{V}) \subseteq (-{}_\flat\Omega_{\max}, {}_\flat\Omega_{\max})$. Since, by definition, $\Phi(\mathcal{V}_0) = \{0\}$ and $\Phi(\mathcal{V}_+) \subseteq (0, {}_\flat\Omega_{\max})$, it follows from Φ's smoothness that $\mathcal{V}_+$'s boundary, $\mathcal{S} = \bar{\mathcal{V}}_+ \setminus \mathcal{V}_+^\circ$, satisfies $\mathcal{S} \subseteq \mathcal{V}_0$. We can thus assume that for any sequence $\{q_i\} \subset \mathcal{V}_+$ such that $q_i \to \mathcal{S}$, it is also true that $\Phi(q_i) \to 0^+$ and $\Phi_W(q_i) \to 0^+$.

4.4.1 The Ultraviolet Limit

We expect that the weight distribution, W, from eq. (23) will direct volumetric flow to all of $\mathcal{V}_+$, but should otherwise be repulsed by the boundary $\mathcal{S}$. Take this

to mean that W should be infinitely flat as $\Phi \to 0^+$. The canonical infinitely flat function is Cauchy's example[29] $\{e^{-1/x^2}$ for $x \neq 0$, 0 for $x = 0\}$, but for our purposes infinitely flat is required only for $x \to 0^+$, and so $e^{-1/x}$ is sufficient.

This suggests that W should include a term $e^{-b/{}_\flat\Phi}$, for some constant b with units of volume. Furthermore, since Ω_ρ is the source of the energetic pressure on Φ, and this pressure is proportional to the scalar energy density ρ, we expect the energetic volume displacement cannot have a granularity finer than the Planck volume ℓ_{P}^3. This implies that b should equal ℓ_{P}^3, and thus, to cover the behavior at points near the boundary, our weight function should follow

$$W \propto e^{-\ell_{\mathrm{P}}^3/{}_\flat\Phi}. \tag{25}$$

Though more generally, we could declare the constant twisted 3-form $\Omega_{\min} > 0$ must exist to set nature's floor for Ω_ρ and Ω_t, write

$$W \propto e^{-\Omega_{\min}/\Phi}\,, \tag{26}$$

and then observe that in our universe

$$\Omega_{\min} = {}^\sharp(\ell_{\mathrm{P}}^3)\,. \tag{27}$$

4.4.2 The Infrared Limit

An additional term is required to describe W's behavior in $\mathcal{V}_+$'s interior, where the volumetric flow should be attracted to regions proportionally with how close Φ is to the vacuum volume limit $\Omega_{\max}$. To provide the required attraction, assume any positive exponential term $e^{f({}_\flat\Phi)}$ (such that f increases polynomially with ${}_\flat\Phi$) mathematically suitable, but for our purposes assume a linear f sufficient. In order for $a \cdot \Phi$ to be the dimensionless scalar that the exponential's argument requires, assume that the linear coefficient a is a constant, positive, 3-form with units of inverse volume, and propose

$$W \propto e^{a\,\Phi} \tag{28}$$

to cover the weighting function behavior at points near its maximum, $\Omega_{\max}$. To ensure consistent scaling regardless of the specific value nature sets for the volumetric limit of the vacuum $\Omega_{\max}$, deduce that $a = 1/\Omega_{\max}$, and write our complete weighting function as

$$W = e^{\Phi/\Omega_{\max}} \cdot e^{-\ell_{\mathrm{P}}^3/{}_\flat\Phi}\,, \tag{29}$$

or, using eq. (26), a general form,

$$W = e^{\Phi/\Omega_{\max}} \cdot e^{-\Omega_{\min}/\Phi} \,. \tag{30}$$

We can now write the local volume displacement from eq. (23) as

$$\frac{\Delta\Omega}{\Delta t} = \frac{\exp\left(\frac{\Phi}{\Omega_{\max}} - \frac{\ell_{\mathrm{P}}^3}{\flat\Phi}\right)}{\int_{\mathcal{V}_+} \sharp \exp\left(\frac{\Phi}{\Omega_{\max}} - \frac{\ell_{\mathrm{P}}^3}{\flat\Phi}\right)} \frac{1}{t_{\mathrm{P}}} \int_{\mathcal{V}_-} (-\Phi) \tag{31}$$

and derive a complete expression for $\partial_t \Omega$.

4.5 Volumetric Flux

For $q \in \mathcal{V}_- \cup \mathcal{V}_0$ where $\Phi \leq 0$, volume flows away at the rate

$$\frac{\partial\Omega}{\partial t} = \frac{1}{t_{\mathrm{P}}} \Phi \tag{32}$$

where the integral across $\mathcal{V}_-$ of $(-\partial_t \Omega)$

$$\sharp \left(\frac{1}{t_{\mathrm{P}}} \int_{\mathcal{V}_-} (-\Phi) \right) \tag{33}$$

is the total **volumetric flux** for the spacelike hypersurface. We sharpen the integral's result so it will remain a 3-form.

For $q \in \mathcal{V}_+$, where $\Phi > 0$ indicates excess volumetric capacity, the volume flows in according to

$$\frac{\partial\Omega}{\partial t} = \frac{\exp\left(\frac{\Phi}{\Omega_{\max}} - \frac{\ell_{\mathrm{P}}^3}{\flat\Phi}\right)}{\int_{\mathcal{V}_+} \sharp \exp\left(\frac{\Phi}{\Omega_{\max}} - \frac{\ell_{\mathrm{P}}^3}{\flat\Phi}\right)} \sharp \left(\frac{1}{t_{\mathrm{P}}} \int_{\mathcal{V}_-} (-\Phi) \right) . \tag{34}$$

Or more succinctly,

$$\frac{\partial\Omega}{\partial t} = \frac{\Phi_W}{\int_{\mathcal{V}_+} \sharp(\Phi_W)} \sharp \left(\frac{1}{t_{\mathrm{P}}} \int_{\mathcal{V}_-} (-\Phi) \right) . \tag{35}$$

This describes a distribution for the universe's expansion rate that is testable against cosmological observations. In the most isolated near-vacuum regions of intergalactic space, where variability in the volumetric potential Φ should be small, eq. (34) predicts an expansion rate that is close to constant across points on the same spacelike hypersurface Σ_t—varying smoothly along timelike trajectories orthogonal to $\mathcal{M}$'s spacelike foliation (fig. 1). While across points on

the same Σ_t, the expansion rate varies in proportion with how long it has been since energy densities were high in the region—which means along the rotational and translational past histories of large cosmological energy concentrations.

Finally, notice that our derivation preserves the continuity of space. We assumed a quantized volumetric flux with the $e^{-\ell_P^3/\flat\Phi}$ term in W. But it was the classical demand that integrals not blow up to infinity[30] that required energy densities to be discrete, and thus led to W's ultraviolet limit. We propose it possible for a manifold to remain smooth while volume is displaced in quanta proportional to the quantized energetic pressure, and thus conclude that our framework is neutral on the question of whether $\mathcal{V}$ is continuous or discrete.

5 Vacuum Spacetime $\mathring{\mathcal{V}}$

In spacetime regions where the energy density ρ approaches vacuum, the volumetric limit Ω_ρ approaches $\Omega_{\max}$ (fig. 8), and for vacuum spacetime the volumetric potential from eq. (20) becomes

$$\Phi = \Omega_{\max} - \Omega_t. \tag{36}$$

To organize investigation of $\mathcal{V}$'s global vacuum state, we partition $\mathring{\mathcal{V}}$ based on the value of Φ, as we did previously with $\mathcal{V}$.

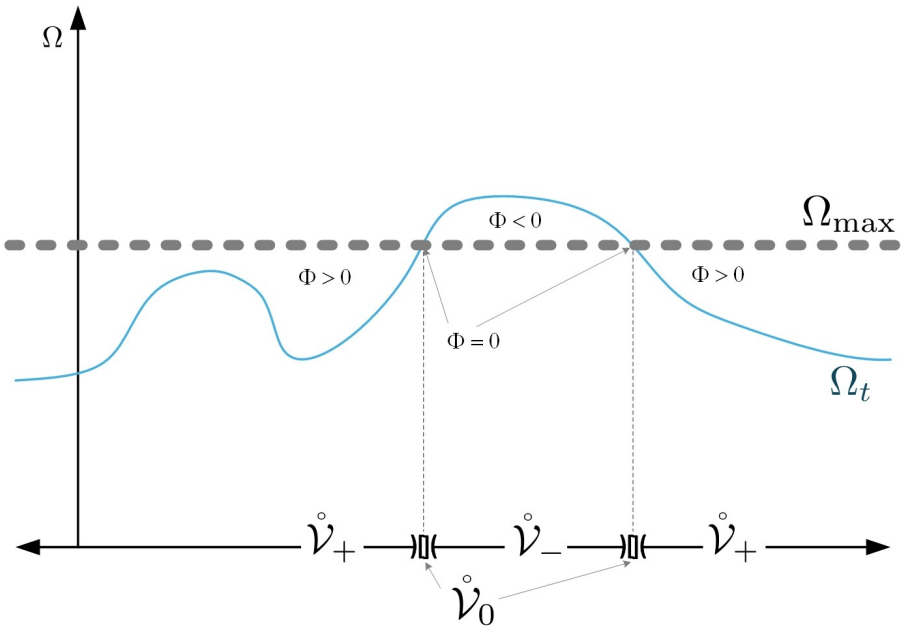


Figure 8: Vacuum Spacetime

5.1 Vacuum State $\mathring{\mathcal{V}}_-$

Definition: $\mathring{\mathcal{V}}_- \equiv \{q \in \mathring{\mathcal{V}} \,|\, {}_\flat\Omega_{\max} < \Omega_t(q)\}$.

This vacuum region defines an overstrained vacuum. We know from eq. (22) that each point $q \in \mathring{\mathcal{V}}_-$ will experience a volumetric outflux equal to

$$\frac{1}{t_{\mathrm{P}}}\,(\Omega_{\max} - \Omega_t) \tag{37}$$

that ensures $\Omega_{t+t_{\mathrm{P}}} = \Omega_{\max}$. As a result, for any non-empty $\mathring{\mathcal{V}}_-$ vacuum state,

$$\mathring{\mathcal{V}}_- \to \emptyset, \;\; \text{as} \;\; \Delta t \to t_{\mathrm{P}}\,. \tag{38}$$

This shows that no region of 3-space can be in this vacuum state for longer than the Planck time at least so long as $\mathring{\mathcal{V}}_+ \neq \emptyset$, and $\int_{\mathring{\mathcal{V}}_+} {}^\sharp\Phi_W$ is sufficiently large to accept the outflux.

5.1.1 When $\mathring{\mathcal{V}} \to \mathring{\mathcal{V}}_-$

For a vacuum spacetime with $\mathring{\mathcal{V}} = \mathring{\mathcal{V}}_-$ (the top line in fig. 9), we would have $\mathring{\mathcal{V}}_+ = \emptyset$ and a contradiction. There would be volumetric outflux at every point, but no region able to accept the flow. This is an example of the previously discussed limit case of eq. (23) where the denominator goes to zero. It represents a spatial fabric stretched so tight that it should tear. We presume a spacetime with $\mathring{\mathcal{V}} = \mathring{\mathcal{V}}_-$ non-physical, or, if physical, representative of a pre-gravitational phase of the volume form Ω_t involving physics beyond the scope of this paper.

5.2 Vacuum State $\mathring{\mathcal{V}}_0$

Definition: $\mathring{\mathcal{V}}_0 \equiv \{q \in \mathring{\mathcal{V}} \,|\, {}_\flat\Omega_{\max} = \Omega_t(q)\}$.

Across these vacuum regions, $\Phi = 0$. There is no volumetric outflux and no capacity for accepting volumetric influx.

5.2.1 When $\mathring{\mathcal{V}} \to \mathring{\mathcal{V}}_0$

A vacuum spacetime with $\mathring{\mathcal{V}} = \mathring{\mathcal{V}}_0$ is stable, but static, with a constant volume of $\Omega_{\max}$ at every point (the middle line in fig. 9). With no sink to receive volumetric flow, any energy inserted would be unable to displace volume, implying that the stress-energy term in Einstein's field equations would be zero everywhere, even when energy is present.

We will call this hypothetical infinitely rigid spacetime, characterized by the global vacuum state $\mathring{\mathcal{V}} = \mathring{\mathcal{V}}_0$, the **null-gravity spacetime**.

5.3 Vacuum State $\mathring{\mathcal{V}}_+$

Definition: $\mathring{\mathcal{V}}_+ \equiv \{q \in \mathring{\mathcal{V}} \,|\, \Omega_{\max} > \Omega_t(q)\}$.

In this vacuum region we have a positive capacity to receive volumetric influx ($\Phi > 0$) everywhere, and no volumetric outflux. This describes a spacetime with innate slack. Volume will deform in response to energetic pressure, at least so long as the inserted energy's total volumetric displacement does not exceed the capacity of $\mathring{\mathcal{V}}_+$.

5.3.1 When $\mathring{\mathcal{V}} \to \mathring{\mathcal{V}}_+$

A vacuum spacetime where $\mathring{\mathcal{V}} = \mathring{\mathcal{V}}_+$ is innately capable of producing gravitational effects, and is thus the anticipated vacuum limit of our spacetime (the bottom line in fig. 9).

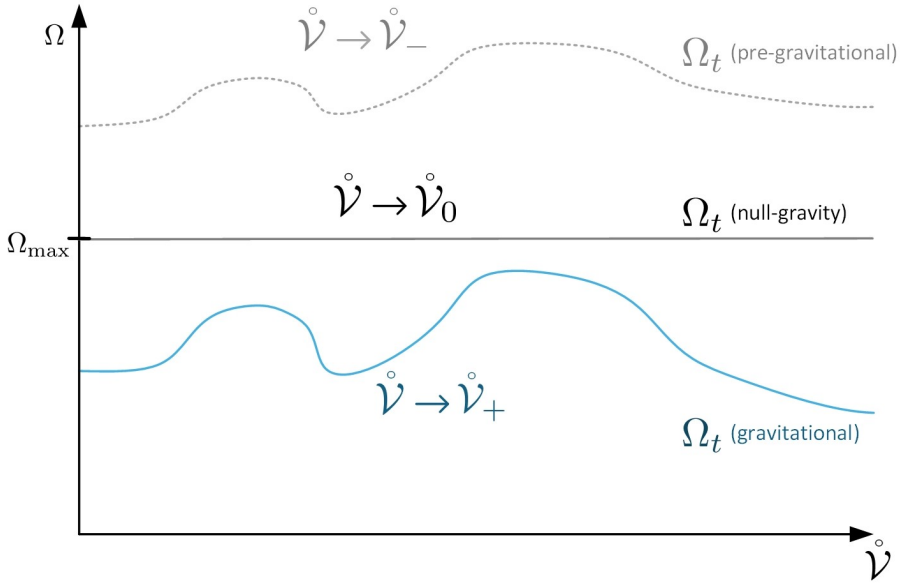


Figure 9: Vacuum States

5.4 Our Vacuum Spacetime

Due to the inherent instability of $\mathring{\mathcal{V}}_-$ shown in eq. (38), we expect $\Omega_t \leq \Omega_{\max}$ to be generically true across our spacetime, and hence propose that the volume of $\mathcal{M}$'s vacuum limit satisfies

$$\mathring{\mathcal{V}} = \overline{\mathring{\mathcal{V}}_+ \cup \mathring{\mathcal{V}}_0}\,. \tag{39}$$

We do allow for the possibility that Ω_t can exceed $\Omega_{\max}$ locally, and hence for $\mathring{\mathcal{V}}_- \neq \emptyset$, though since eq. (38) shows that no point of spacetime can be an element of $\mathring{\mathcal{V}}_-$ for longer than the Planck time, we propose that $\mathring{\mathcal{V}}_-$ is nowhere dense, and that its instability may relate to the quantum vacuum fluctuations.

5.5 The Homogeneous Vacuum

To understand the homogeneous vacuum, we imagine the limit where $\mathring{\mathcal{V}}$'s volume form Ω_t approaches constant value $\mathring{\Omega}$. But first we will establish whether it makes physical sense for Ω_t to have the same volume across all of $\mathcal{V}$.

5.5.1 Compactness of $\mathcal{V}_+ \cup \mathcal{V}_-$

If we can correlate the influx eq. (34) to the observed cosmological distribution of dark energy, we will have direct observational evidence of volumetric flow. This will demonstrate that both the influx equation's denominator and its total gradient are finite and non-zero. We know from section 4.3.1 that a finite gradient demonstrates that the amount of energy causing volumetric distortions across the spacelike hypersurface is finite. A measurable gradient demonstrates that the portion of the hypersurface capable of receiving volumetric flux, $\mathcal{V}_+$, is also finite. Together these demonstrate that the portion of the universe which contains energy as well as that which is capable of responding to volumetric pressure from energy, are both finite and compact.

5.5.2 Compactness of $\mathcal{V}_0$

If there exists a homogeneous vacuum across which every point has the same volume, then we can assume the existence of a constant twisted 3-form $\mathring{\Omega}$ which satisfies eq. (10) and simplifies its expression to

$$\mathbb{V} = \int_{\mathcal{V}} \mathring{\Omega} = {}_\flat\mathring{\Omega} \int_{\mathcal{V}} \omega_0 \,. \tag{40}$$

It would then follow that $\mathbb{V}$ is finite only if $\mathcal{V}$ is compact. With the compactness of both $\mathcal{V}_+$ and $\mathcal{V}_-$ established, the only open we must resolve before assuming the existence of $\mathring{\Omega}$ is the question of $\mathcal{V}_0$'s compactness. Phrased differently, the open question is whether we can assume compactness for that portion of the universe where there is no energy and no ability to be impacted by energy's volumetric pressure—and hence where nothing energetic can happen aside from quantum background fluctuations. We defer to Archimedes' Sandreckoner [31] and propose this remaining inactive portion of the universe to be compact. We reason that with the finiteness of the totality of the energetic universe established, the alternative proposition—that the portion of the universe devoid of energetic interactions is infinite—is an extraordinary claim that lacks sufficient supporting evidence.

Our hypotheses to this point remain sound without this assumption. Full compactness though ensures the existence of a valid state for our vacuum spacetime with constant volume everywhere, which we can then use as our homogeneous vacuum limit.

5.5.3 Homogeneous Vacuum Volume

We define the homogeneous vacuum to be not only the limit where $\rho \to 0$ but also where Ω_t approaches the **homogeneous vacuum volume** $\mathring{\Omega}$ at all points of $\mathcal{V}$. In addition, since $\mathring{\Omega}$ must obey the same relations as Ω_t, we can say that the set of physically meaningful choices for $\mathring{\Omega}$ is constrained by

$$0 < {}^{\sharp}(\ell_{\mathrm{P}}^3) < \mathring{\Omega} < \Omega_{\max} \,. \tag{41}$$

Finally, since $\mathring{\Omega} = \Omega_{\max}$ represents section 5.2's null-gravity spacetime, we can conclude that across the class of gravitationally meaningful spacetimes for $\mathcal{M}$, as $\mathring{\Omega} \to \Omega_{\max}$, the homogeneous vacuum $\mathring{\mathcal{V}}$ will approach the null-gravity spacetime where $\mathring{\mathcal{V}}_+$ and $\mathring{\mathcal{V}}_-$ are both empty sets and $\mathring{\mathcal{V}} = \mathring{\mathcal{V}}_0$. It is obvious that the null-gravity spacetime is also approached as $\mathring{\Omega} \to {}^{\sharp}(\ell_{\mathrm{P}}^3) = \Omega_{\min}$, but since our paper will not use the result we will leave the observation unproven.

6 Gravitation

A full account of how the field equations of general relativity relate to this framework is for future work. For now, we show how the gravitational coupling relates to the framework and derive some immediate implications.

6.1 Gravitational Coupling

The observation that $\mathcal{V}$ approaches the null-gravity spacetime as $\mathring{\Omega} \to \Omega_{\max}$ in section 5.5.3 yields the important conclusion that a given spacetime's ability to deform in response to energy, and hence, the base strength of the gravitational coupling $\mathring{G}$, is proportional to the innate volumetric slack, $\Omega_{\max} - \mathring{\Omega}$, of its homogeneous vacuum volume, $\mathring{\mathcal{V}}$, and thus

$$\mathring{G} \propto \int_{\mathring{\mathcal{V}}} \left(\Omega_{\max} - \mathring{\Omega} \right) . \tag{42}$$

For a non-vacuum spacetime, we can replace $\Omega_{\max}$ with Ω_ρ, and $\mathring{\Omega}$ with Ω_t and propose

$$G_t \propto \int_{\mathcal{V}} \left(\Omega_\rho - \Omega_t \right) \tag{43}$$

for energetic spacetime's gravitational coupling. Inserting from eq. (20), we get

$$G_t \propto \int_{\mathcal{V}} \Phi \,, \tag{44}$$

and conclude that the energetic gravitational coupling is proportional to the universe's global volumetric potential.

6.2 Energetic Capacity

From eq. (43), we see that a second null-gravity bound for $\mathcal{V}$ occurs if the volumetric limit from eq. (17) equals the available volume, and thus $\Omega_\rho = \Omega_t$ everywhere. As with section 5.2's null-gravity spacetime (where $\mathring{\mathcal{V}}_+$ is empty), if no point in energetic spacetime has excess volumetric potential, then $\mathcal{V}_+$ will be empty, and $\mathcal{V}$ will have no capacity for accepting volumetric influx.

For a volume that has reached its energetic capacity, we can substitute eq. (17) into $\Omega_\rho = \Omega_t$ and get

$$\zeta \circ \rho_{\text{cap}} = {}_\flat\Omega_t \,. \tag{45}$$

Our proposition in section 4 that the energy-volume relation ζ be smooth and strictly monotonic ensures the existence of an inverse. We can thus write

$$\rho_{\text{cap}} = \zeta^{-1} \circ {}_\flat\Omega_t \tag{46}$$

and integrate across the total volume to produce the **energetic capacity** of a volume form Ω_t, given by

$$\int_{\mathcal{V}} \rho_{\text{cap}} \cdot \Omega_t = \int_{\mathcal{V}} \left(\zeta^{-1} \circ {}_\flat\Omega_t\right) \cdot \Omega_t \,. \tag{47}$$

If the total energy of the universe $\mathbb{E}$ becomes equal to or greater than the energetic capacity, then space becomes infinitely rigid as $\mathcal{V}_+ \to \emptyset$, and $G_t \to 0$. We conclude that for spacetime to exhibit gravitational effects, its total energy must remain bounded by

$$\mathbb{E} < \int_{\mathcal{V}} \left(\zeta^{-1} \circ {}_\flat\Omega_t\right) \cdot \Omega_t \,. \tag{48}$$

For the homogeneous vacuum, this reduces to

$$\mathbb{E} < \zeta^{-1}({}_\flat\mathring{\Omega}) \int_{\mathcal{V}} \mathring{\Omega} \,, \tag{49}$$

We can then substitute in from eq. (40), and get an expression for the energetic capacity of the homogeneous vacuum that depends only on the total volume $\mathbb{V}$, the homogeneous vacuum volume $\mathring{\Omega}$, and the energy-volume relation ζ.

$$\frac{\mathbb{E}}{\mathbb{V}} < \zeta^{-1}({}_\flat\mathring{\Omega})\,. \tag{50}$$

6.3 A Brief Investigation of ζ

We have specified throughout this paper that both the energy-volume relation ζ and the volume form Ω_t are strictly positive and smooth. Though as discussed in section 4.3.1, we can make a stronger statement regarding positivity and write

$$\Omega_t \geq {}^\sharp(\ell_{\mathrm{P}}^3) > 0\,.$$

More importantly, to prevent volumetric flow from driving Ω_t to a non-positive value, Φ must be non-negative whenever Ω_t equals its minimum $\Omega_t = {}^\sharp(\ell_{\mathrm{P}}^3)$, from which

$$\zeta \geq \ell_{\mathrm{P}}^3 > 0$$

follows.

Though we expect ζ's full form to have non-linear terms, we start by exploring a linear ζ. The strictly monotonically decreasing property lets us derive the characteristic density, or linear scale, ρ_*, of the energy-volume relation through $\zeta(\rho_*) = \ell_{\mathrm{P}}^3$. Solving for this condition in the linear case yields

$$\zeta(\rho)/{}_\flat\Omega_{\max} = 1 - \frac{1 - \ell_{\mathrm{P}}^3/{}_\flat\Omega_{\max}}{\rho_*}\,\rho + \mathcal{O}(\rho^2)\,. \tag{51}$$

Though, since the $\ell_{\mathrm{P}}^3/{}_\flat\Omega_{\max}$ term in the linear coefficient's numerator is multiple orders of magnitude smaller than the Planck volume and thus indistinguishable from zero, we can safely ignore the term and write

$$\zeta(\rho)/{}_\flat\Omega_{\max} = 1 - \frac{1}{\rho_*}\,\rho + \mathcal{O}(\rho^2)\,. \tag{52}$$

We use this form for ζ to highlight the obvious parallels with eq. (11) from general relativity. While our linear coefficient is $1/\rho_*$ rather than the $(8\pi/15)\cdot(r^2G/c^2)$ of general relativity, it is important to recognize that we cannot directly equate the two. Equation (11) is a series expansion in the radius around a point, while eq. (52) is a series expansion in the energy density around the vacuum.

6.3.1 A Closed Form

Conjecture: The energy volume relation ζ is infinitely flat as $\rho \to \infty$, with form

$$\zeta(\rho) = ({}_\flat\Omega_{\max} - \ell_{\mathrm{P}}^3) \cdot e^{-\rho/\rho_*} + \ell_{\mathrm{P}}^3 \,, \tag{53}$$

which is equivalent to

$$\zeta(\rho) = {}_\flat\Omega_{\max} \cdot e^{-\rho/\rho_*} \tag{54}$$

except in regions of space where $\rho \gg \rho_*$.

With this form for ζ, we can derive its inverse

$$\zeta^{-1}(v) = \rho_* \cdot \ln\left(\frac{{}_\flat\Omega_{\max}}{v}\right) \tag{55}$$

and get a final form for the energetic capacity of the homogeneous vacuum from eq. (50)

$$\frac{\mathbb{E}}{\mathbb{V}} < \rho_* \cdot \ln\left(\frac{\Omega_{\max}}{\mathring{\Omega}}\right) . \tag{56}$$

Similarly, we get updated expressions for the volumetric potential from eq. (20),

$$\Phi = \Omega_{\max} \cdot e^{-\rho/\rho_*} - \Omega_t \,, \tag{57}$$

and the gravitational coupling from eq. (44),

$$G_t \propto \int_{\mathcal{V}} \left(\Omega_{\max} \cdot e^{-\rho/\rho_*} - \Omega_t\right) . \tag{58}$$

6.3.2 Singularities

While numerically unimportant, the last ℓ_{P}^3 term in eq. (53) has theoretical importance. It ensures that the energy volume relation asymptotically approaches the Planck volume rather than 0. This protects Ω_t's positivity, and excludes the possibility of gravitational singularities occurring on the manifold. Though it does not have exclude the possibility of an event horizon. The conjecture also aligns precisely with our preceding derivation, as eq. (52) is the first order Taylor series approximation of eq. (54).

7 Future Work

Numerical and Experimental Verification

There is a wealth of astronomical data available against which we can test our framework's volume conservation mechanics. First, is to verify that when the universe is in a state where $\partial_t^2 \Omega \approx 0$ that there exists a solution for eq. (34)'s $\partial_t \Omega$ that reproduces the appearance of an elastic spacetime at the scales of the energy densities of our own solar system. This will be the topic of our second paper.

Relevant to dark matter, the spatial offset observed between the Bullet Cluster's baryonic center of mass and its gravitational well[32, 33] could be a natural consequence of volumetric flow in the wake of a violent collision that significantly shifts a cluster's center of mass away from the center of its residual stress. Similarly, numerical simulations of $\partial_t \Omega$ from section 4.5 could be tested against the Milky Way rotation data from the European Space Agency's Gaia satellite[34–36].

Relevant to dark energy, the Dark Energy Spectroscopic Instrument's mapping of baryon acoustic oscillations[37, 38] could have signatures consistent with eq. (34).

General Relativity

An examination of Einstein's papers from the period between special and general relativity reveals early attempts to frame gravity as a scalar field conceptually similar to our framework[39]. The scalar field was set aside because the full richness of relativistic mechanics requires the metric tensor. However, our framework suggests that the idea of an underlying scalar field has merit and can coexist with the metric tensor in a full theory of gravity, by placing the volume 1-density, Ω_t, at the foundation and presuming that the conformal metric, $\bar{\gamma}_{ij}$, rests atop it.

With respect to existing relativity theory, the adjustment we make is to replace spacetime's perfect elasticity with a locally plastic medium which continuously reflattens over time through a global volume conservation process analogous to the mechanical annealing of amorphous metallic glasses[40–42]. Aside from those cases where future investigation reveals that current theory's topological assumptions are inconsistent with the volume conservation mechanics, we propose that relativistic dynamics in this framework behave as current theory dictates. For example, we expect that gravitational waves will propagate with the same volume-preserving shear that current theory predicts, while future work on the residual stress implications may reveal higher order divergences from existing theory in the wave dynamics.

Significant theoretical work remains to complete a bridge to the tensor calculus of Einstein's field equations. Particularly interesting is eq. (54)'s impact on cosmic inflation and black hole dynamics.

Quantum Field Theory

We believe that conjoining $\mathcal{Q}$ with 3-space by considering all points on the same spacelike hypersurface as the singular neighborhood of an indiscrete topological substrate, can serve as a useful model for framing the fields of quantum theory. A first proposition is that ${}_\flat\Omega_t$ is a scalar quantum field which provides coupled particles with the scale that $\bar{\gamma}_{ij}$ lacks, and that the rate at which Ω_t samples Ω_ρ's energetic pressure provides the tick of the Planck time. The sampling interval t_{P} and the local reference for the Planck scale ℓ_{P} then combine to determine light's speed $c = \ell_{\mathrm{P}}/t_{\mathrm{P}}$.

8 Acknowledgements

I am especially thankful to Henk Neefs for multiple inspiring discussions over breakfast in the Intel RNB Cafe. Additional thanks to Janna Levin's *How the Universe got its Spots*[43] for its characterization of late 1990s UC Berkeley physics culture. Reading it for the first time in 2026 stirred memories of my own time as an undergraduate in the physics department earlier the same decade. It was then that I first formed the thought that there must be something about the topological structure of space we are missing. A return to that time with the perspective of hindsight, and with my own poetic ruminations on the topic fully formed[44], is where the current paper started.

9 AI Usage Note

Generative AI (Google Gemini) was used during the drafting of this paper to cross-reference historical literature, identify parallels with related work, check long derivations for dimensional errors, and scrub the use of notation against contemporary physics conventions. All theoretical concepts, mathematical derivations, logical deductions, and final manuscript contents are the sole work and responsibility of the author.

References

1. Einstein A (1905) Über einen die erzeugung und verwandlung des lichtes betreffenden heuristischen gesichtspunkt [on a heuristic point of view concerning the production and transformation of light]. Annalen der Physik 322:132–148

2. Einstein A (1909) Über die entwicklung unserer anschauungen über das wesen und die konstitution der strahlung [on the development of our views concerning the nature and constitution of radiation]. Physikalische Zeitschrift 10:817–825

3. Broglie L de (1925) On the theory of quanta. Annales de Physique 10:22–128. https://doi.org/10.1051/anphys/192510030022

4. Davisson C, Germer LH (1927) Diffraction of electrons by a crystal of nickel. Phys Rev 30:705–740. https://doi.org/10.1103/PhysRev.30.705

5. Shibata M, Nakamura T (1995) Evolution of three-dimensional gravitational waves: Harmonic slicing case. Phys Rev D 52:5428–5444. https://doi.org/10.1103/PhysRevD.52.5428

6. Baumgarte TW, Shapiro SL (1998) On the numerical integration of einstein's field equations. Phys Rev D 59:024007. https://doi.org/10.1103/PhysRevD.59.024007

7. Greene B, Kabat D, Levin J, Porrati M (2026) Compactification without orientation, or a topological scenario for CP violation. Physical Review D 113:065020. https://doi.org/10.1103/fp2t-9xwj

8. Greene B, Kabat D, Levin J, Porrati M (2026) Klein bottle cosmology. Physical Review D 113:063576. https://doi.org/10.1103/dyym-ywsr

9. Wheeler JA (1980) Pregeometry: Motivations and prospects. In: Marlow AR (ed) Quantum theory and gravitation. Academic Press, New York, pp 1–11

10. Misner CW, Thorne KS, Wheeler JA (1973) Gravitation. W. H. Freeman

11. Frankel T (2011) The geometry of physics: An introduction, 3rd ed. Cambridge University Press

12. Nakahara M (2003) Geometry, topology and physics, 2nd ed. CRC Press

13. Aspect A, Dalibard J, Roger G (1982) Experimental test of bell's inequalities using time-varying analyzers. Phys Rev Lett 49:1804–1807. https://doi.org/10.1103/PhysRevLett.49.1804

14. Weihs G, Jennewein T, Simon C, et al (1998) Violation of bell's inequality under strict einstein locality conditions. Phys Rev Lett 81:5039–5043. https://doi.org/10.1103/PhysRevLett.81.5039

15. Hensen B et al (2015) Loophole-free bell inequality violation using electron spins separated by 1.3 kilometres. Nature 526:682–686. https://doi.org/10.1038/nature15759

16. Giustina M et al (2015) Significant-loophole-free test of bell's theorem with entangled photons. Phys Rev Lett 115:250401. https://doi.org/10.1103/PhysRevLett.115.250401

17. Shalm LK et al (2015) Strong loophole-free test of local realism. Phys Rev Lett 115:250402. https://doi.org/10.1103/PhysRevLett.115.250402

18. Steinberg AM, Kwiat PG, Chiao RY (1993) Measurement of the single-photon tunneling time. Phys Rev Lett 71:708–711. https://doi.org/10.1103/PhysRevLett.71.708

19. Sainadh US, Xu H, Wang X, et al (2017) Attosecond angular streaking and tunnelling time in atomic hydrogen

20. Ramos R, Spierings D, Racicot I, Steinberg AM (2020) Measurement of the time spent by a tunnelling atom within the barrier region. Nature 583:529–532. https://doi.org/10.1038/s41586-020-2490-7

21. Einstein A (2009) Quantum theory at the crossroads: Reconsidering the 1927 solvay conference. In: Bacciagaluppi G, Valentini A (eds). Cambridge Univ. Press, pp 440–442

22. Einstein A, Podolsky B, Rosen N (1935) Can quantum-mechanical description of physical reality be considered complete? Phys Rev 47:777–780. https://doi.org/10.1103/PhysRev.47.777

23. Bell JS (1964) On the einstein podolsky rosen paradox. Physics Physique Fizika 1:195–200. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195

24. Maldacena J, Susskind L (2013) Cool horizons for entangled black holes. Fortsch Phys 61:781–811. https://doi.org/10.1002/prop.201300020

25. Gray A, Vanhecke L (1979) Riemannian geometry as determined by the volumes of small geodesic balls. Acta Math 142:157–198. https://doi.org/10.1007/BF02395060

26. Feoli A (2018) Plasticity of spacetime as an alternative to dark matter. Mod Phys Lett A 33:1850194. https://doi.org/10.1142/S021773231850194X

27. Dyson FW, Eddington AS, Davidson C (1920) A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of may 29, 1919. Philosophical Transactions of the Royal Society of London Series A 220:291–333. https://doi.org/10.1098/rsta.1920.0009

28. Bertotti B, Iess L, Tortora P (2003) A test of general relativity using radio links with the cassini spacecraft. Nature 425:374–376. https://doi.org/10.1038/nature01997

29. Cauchy A-L (1822) Mémoire sur le développement des fonctions en séries, et sur l'intégration des équations différentielles linéaires d'un ordre quelconque [memoir on the expansion of functions in series]. Bulletin des Sciences, par la Société Philomathique de Paris 49–54

30. Planck M (1901) Ueber das gesetz der energieverteilung im normalspectrum [on the law of distribution of energy in the normal spectrum]. Annalen der Physik 309:553–563. https://doi.org/10.1002/andp.19013090310

31. Archimedes (1956 [original: 340 BCE]) The sand reckoner. In: Newman JR (ed) The world of mathematics vol. 1. Simon; Schuster, pp 420–429

32. Clowe D, Bradač M, Gonzalez AH, et al (2006) A direct empirical proof of the existence of dark matter. The Astrophysical Journal Letters 648:L109–L113. https://doi.org/10.1086/508162

33. Markevitch M, Gonzalez AH, Clowe D, et al (2004) Direct constraints on the dark matter self-interaction cross section from the merging galaxy cluster 1E 0657-56. The Astrophysical Journal 606:819–824. https://doi.org/10.1086/383158

34. Jiao Y, Hammer F, Wang H, et al (2023) Detection of the Keplerian decline in the Milky Way rotation curve. Astronomy & Astrophysics 678:A208. https://doi.org/10.1051/0004-6361/202347513

35. Moffat JW (2024) Implications of the Milky Way Declining Rotation Curve. arXiv e-prints

36. Melchiorri A et al (2026) The Rotation Curve of the Milky Way: State of the Art, the Keplerian decline debate, and implications for dark matter. arXiv e-prints

37. Adame AG et al (2024) DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations. arXiv e-prints

38. DESI Collaboration (2025) Dynamical Dark Energy in light of the DESI DR2. arXiv e-prints

39. Einstein A (1912) Lichtgeschwindigkeit und statik des gravitationsfeldes [the speed of light and the statics of the gravitational field]. Annalen der Physik 343:355–369. https://doi.org/10.1002/andp.19123430204

40. Dreyer O (2012) Internal relativity. Foundations of Physics 42:1346–1361. https://doi.org/10.1007/s10701-012-9658-0

41. Dreyer O (2008) Early universe cosmology in internal relativity. arXiv preprint

42. Spaepen F (1977) A microscopic mechanism for steady state inhomogeneous flow in metallic glasses. Acta Metallurgica 25:407–415. https://doi.org/10.1016/0001-6160(77)90232-2

43. Levin J (2002) How the universe got its spots: Diary of an unobservable universe. Princeton University Press, Princeton, NJ

44. Chamberlain JD (2025) Symmetry emergent